A HIGHLY INTERACTIVE DISCOURSE STRUCTURE

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INTRODUCTION

This somewhat speculative chapter is grounded in observations made during the detailed analysis of two very different mathematics lessons. The first is a high school mathematics/physics lesson conducted by Jim Minstrell toward the beginning of the school year. In broadest terms, the question explored by Minstrell's class is how to determine the "best value" for some quantity when a number of measurements have been taken. The day before the lesson examined here, Minstrell had posed the question in terms of five different measurements of someone's blood alcohol content. Eight students had also measured the width of a table, obtaining a range of different values. On this fourth day of the school year the students discuss whether some or all of the numbers should be taken into account, and how best to combine them. During the lesson, Minstrell's questioning style invites contributions from the students. These contributions provide a significant proportion of the content of the lesson.

The second lesson to be examined occurs in Deborah Ball's third grade mathematics classroom, in the middle of the school year. Ball's students have been discussing the properties of even and odd numbers. The previous day they had met with a class of fourth graders to discuss some of the issues they had been grappling with – for example, is the number zero even, odd, or "special"? Ball begins this day's lesson with the request that the students reflect on their thinking and learning, using the previous day's meeting as a catalyst for

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reflection. The ensuing discussion takes on a life of its own, with an intermingling of discussions of content and reflections on student learning.

In some ways the two lessons discussed in this chapter are worlds apart. To begin with the obvious, students in elementary and high school are very different in terms of social and cognitive development. In Ball's class the subject matter content is elementary mathematics, and the agenda is to have students reflect on their understandings. In Minstrell's class the subject matter is more advanced, and the agenda is to have the students sort out how best to make sense of it. Thus the agendas are radically different. Moreover, the two classroom communities are at very different points in their evolution. At the beginning of the year, Minstrell's class has not yet been shaped as a functioning discourse community (that is, the norms of interaction have not been established and internalized). By mid-year, Ball's class has well established sociomathematical norms.

In other ways, these two lessons are very similar. Both Minstrell and Ball work very hard to have their classrooms function as communities of disciplined inquiry. A major instructional goal is for students to experience mathematics/physics as a sense-making activity – as a disciplined way of understanding complex phenomena. A long-term goal of both teachers is for their students to internalize this form of sense-making. They believe it is important for their students to see themselves as people who are capable of making sense of mathematical and real-world phenomena, by reasoning carefully about them. Part of the way that Ball and Minstrell work toward these goals is to have their classrooms function as particular kinds of discourse communities, in which inquiry and reflection are encouraged and supported. Over the course of the year, sociomathematical norms in support of such practices are established. Classroom discourse practices support students' engagement with the content and their reflection on both the content and their understandings of it. One such discourse practice, captured as a pedagogical routine, is the focus of this paper.

This chapter unfolds as follows. I begin with a brief description of the analytic enterprise that gave rise to the discussion in this chapter, the work of the Teacher Model Group at Berkeley. This discussion explains how we came to examine lessons by Minstrell and Ball, and some of what we saw – including the classroom routine that I claim is common to both teachers. I also point to some of the literature on classroom discourse practices, to establish the contrast between traditional discourse patterns and the highly interactive routine used by Ball and Minstrell. With this as context, I move to a description of the routine itself. Following the general description, I work through sections of lessons by Minstrell and Ball, showing in detail how this routine plays out in practice. In a concluding discussion, I elaborate on a conjecture that this routine serves as a mechanism that teachers can use to help their classrooms evolve into highly interactive communities of inquiry.

BACKGROUND AND CONTEXT

The work described here is part of an ongoing body of work conducted by the Teacher Model Group (TMG) at Berkeley. In broadest terms, the goal of the TMG is to provide a rigorous theoretical characterization of the teaching process, employing an analytical framework that explains how and why teachers make the choices they do, in the midst of classroom interactions. Roughly speaking, the idea is that teachers' decision-making is a function of their goals, beliefs, and knowledge. That is, a teacher enters a specific classroom with certain (content-related and social) goals in mind for that day, as well as overarching goals for the school year. That teacher has certain understandings or beliefs about the nature of mathematics, about appropriate teaching practices, and about his or her students. He or she has various kinds of knowledge as well – knowledge of the mathematics, of pedagogy in general, of the students in the class, about the ways that class has unfolded in recent days and where the teacher wants it to go, etc. During the lesson, various things come up. For example, a student may make a mistake, and the teacher may suspect that other students need help with the same concept. Or, a student who has been quiet may risk a suggestion. Any of a million things may happen. How will the teacher respond, and why?

According to the theory, what the teacher does depends on the teacher's knowledge, goals, and beliefs. Take the case of a student saying something incorrect. How serious does the teacher consider this mistake to be? Does the teacher believe mistakes should be dealt with immediately? Does he or she believe in "correcting" mistakes, or in seeking the underlying cause for them? How much time does the teacher have to deal with the issue? What pedagogical methods or classroom routines does the teacher have available for dealing with this situation? On the basis of all of these, the teacher will choose whether or not to address the issue. How the issue is pursued will depend on what options the teacher perceives are available, what the costs and benefits of each option might be, and what the constraints of the situation might allow.

This brief description merely suggests a research agenda, which has unfolded over more than a decade (see, e.g. Schoenfeld, 1998, 1999, for details). That agenda has theoretical components (what do we mean by knowledge, goals, and beliefs? How do they interact?) and a corresponding body of empirical work, in which the theory is used to build models of specific teachers teaching specific lessons. The models serve to test the adequacy and scope of the theory.

Part of the specification of the model of a teacher teaching a particular lesson is the delineation of the cognitive and interactional resources that are available to the teacher and relevant to the lesson being modeled. Here I will not describe the architecture of knowledge used in the model, save to say that TMG's assumptions regarding the organization of memory are consistent with the standard cognitive model. Rather, I will focus on one particular kind of interaction, the classroom routine. As Leinhardt notes,

> Routines are vital. They reduce the cognitive processing load for both the student and the teacher; they are easy to teach because, by second grade, students have a schema of "learn the routine for *X*" – they expect them. Routines are considered efficient when they elicit an action with a minimum of time and confusion. Effective teachers have management, support, and exchange routines in place by the end of the second day in a school year. They retain 90% of these routines at midyear (Leinhardt, Weidman & Hammond, 1987). But routines are also subtle and set the tone of the class (Leinhardt, 1993, p. 15).

One classic teaching routine, a nearly ubiquitous discourse structure in classrooms in the U.S., is the "IRE sequence" – a sequence in which a teacher initiates an interaction, the student responds, and the teacher evaluates the response (see, e.g. Cazden, 1986; Mehan, 1979; Sinclair & Coulthard, 1975). This structure can be implemented with a fair amount of latitude, in that the student response and the teacher's evaluation of it can range from a word or a phrase to lengthy expositions. However, the stereotype – grounded in reality – is that in traditional didactic mathematics lessons, short IRE sequences are ideal vehicles for fostering student mastery of procedural skills. Typically, at some point in a lesson a teacher will ask students to provide their answers to a set of assigned problems. Students will be called upon to give their answers to the problems in sequence, and the teacher will assess the responses, possibly elaborating on points of importance.

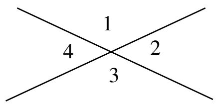
Here, as an example, is part of the dialogue from a U.S. lesson on complementary and supplementary angles (U.S. Department of Education, 1997). The lesson comes from the videotape collection of the Third International Mathematics and Science Study (TIMSS). The tapes that were publicly released were chosen because of their representativeness.

The teacher begins the lesson by going over a homework assignment. After reminding students that measures of complementary angles add up to ninety degrees, he calls on a series of students to give their answers to the problems. The teacher works through the first problem with a student who had not done the assignment, and then continues:

I1. Teacher: What's the complement of an angle of seven degrees? Ho.
R1. Student: Eighty-three degrees.
E1. Teacher: Eighty-three.
I2. Teacher: The complement of an angle of eighty-four, Lindsay?
R2. Student: Sixteen.
E2/I3. Teacher: You sure about your arithmetic on that one?
R2. Student: Oh. Six.
E3. Teacher: Six. Six degrees.
I4. Teacher: Albert, number four.
R4. Student: Seventy-nine degrees?
E5. Teacher: [acknowledges correctness by continuing].
I6. Teacher: Number five, Joey.
R6. Student: Thirty-three.
E6/I7. Teacher: Sure about that? Claudia?
R7. Student: Twenty-three.
E7. Teacher: Twenty-three. You've got to be careful about your arithmetic . . .

Later in the lesson the teacher introduces the students to supplementary and vertical angles. The relevant information for working on the problems he assigns is that vertical angles are equal, and that supplementary angles add up to one hundred eighty degrees. After handing out a work sheet, the teacher continues:

Teacher: Look at the examples on the top. Similar to your warm-up. Look at the figure [below]. . . . Find the measure of each angle.



I8. Teacher: If angle three is one hundred twenty degrees . . . and angle three and angle one are vertical, what must angle one be equal to?
R8. Student: One twenty.
E8. Teacher: One hundred twenty degrees.
I9. Teacher: What can you tell me about angles two and three?
R9. Student: That they are vertical.
E10. Teacher: Two and three are not vertical. One and three are vertical. Two and four are vertical. Two and three are supplementary.
I11. Teacher: So, if three is a hundred and twenty, what must two be equal to?
R11. Student: Sixty.
E11. Teacher: Sixty. Two is sixty.
I12. Teacher: What must four be equal to?
R12. Student: Sixty.
E12: Teacher: Okay.

Little needs be said here by way of analysis. In terms of discourse, the I, R, and E labels say it all. The teacher posed a series of "short answer" questions.

When students responded correctly he confirmed the correctness of their answers. When they responded incorrectly he had them re-calculate (in E2/I3 and E6/I7) or, in the case of a factual mistake (E10), he informed them of the correct answer. In terms of sociomathematical norms, an earlier observation made by Lampert hits the nail on the head:

> Commonly, mathematics is associated with certainty; knowing it, with being able to get the right answer, quickly . . .. These cultural assumptions are shaped by school experience, in which *doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher. Beliefs about how to do mathematics and what it means to know it in school are acquired through years of watching, listening, and practicing (Lampert, 1990, p. 32).

As noted in the introduction, both Minstrell and Ball have very different goals for their students than the outcomes of traditional instruction described above by Lampert. Rather serendipitously, the TMG wound up analyzing lessons by both Minstrell and Ball. The Minstrell study came about early in TMG work. The first lesson we analyzed (see Zimmerlin & Nelson, 1999) was of a student teacher teaching a rather traditional lesson. While we were engaged in that analysis Emily van Zee, who had worked with Minstrell, joined the group. At that time van Zee was working on the analysis of a lesson taught by Minstrell. The van Zee and Minstrell (1997) analysis focused on a questioning strategy employed by Minstrell, "reflective tosses." TMG believed it would be useful to do a complementary analysis, focusing on Minstrell's knowledge, goals, and beliefs. In addition, Minstrell's lesson was very different from the lesson we had been analyzing. Minstrell was an experienced teacher, while Nelson was a beginner. Minstrell's lesson was non-traditional and of his own design, while Nelson's was traditional. And, while unexpected events in Nelson's lesson had caused him to run into some difficulties, unexpected events in Minstrell's lesson were dealt with smoothly. Studying Minstrell's lesson would be good for theory building: examining radically different cases is an important way to test the scope of an emerging theory, as well as its robustness.

Over a period of about two years, TMG refined its understanding of the Minstrell lesson and constructed a model of Minstrell's teaching (of that lesson). One component of the model was an interactive routine used by Minstrell to solicit ideas and information from his students. This routine, which used Minstrell's "reflective tosses," was a powerful tool for enfranchising the students. It made use of their ideas, rather than information provided by the teacher, to deal with the issues at hand.

Success in modeling the Minstrell lesson led to some confidence about the robustness of the TMG's theoretical constructs. Then, as the model of the lesson was being refined (see Schoenfeld, Minstrell & van Zee, 1999, for a description of the model), members of the research group saw a videotape of Deborah Ball's "Shea numbers" class. This lesson offered new challenges. Although Minstrell's and Nelson's lessons are very different, they share some very important properties. They deal with high school mathematics (and thus with high school students). And, both lessons are driven by the teacher's agenda. In Ball's class the students are third graders, so there are significant differences in terms of the students' knowledge bases, and their cognitive and social development. Equally important, the lesson in question had taken unexpected twists and turns. The agenda appeared to be co-constructed by the students and teacher, in response to ongoing events. The question was, could TMG's theoretical notions suffice to model this lesson – or was a detailed model of this lesson beyond the scope of the theory?

For quite some time the issue was in doubt; in Schoenfeld, Minstrell, and van Zee (1999) the authors noted that they had, thus far, been unsuccessful in modeling Ball's decision-making during the lesson in question. Ultimately, however, a model of the first part of the lesson, with all its unexpected twists and turns, was developed. When the structure of the lesson came to be understood, Ball's decision-making was represented in flow-chart form. At that point, TMG made a surprising discovery. The decision procedure represented by the flow chart was remarkably similar to the decision procedure that we had attributed to Minstrell!

The classroom routine represented by that decision-making structure is the focus of this chapter. I conjecture that this routine occurs with some frequency in "inquiry-oriented" classrooms, and that it helps such teachers to establish classroom communities in which disciplined inquiry is a major feature. The following section of this chapter provides a description of the routine.

A COMPLEX ROUTINE FOR SOLICITING AND WORKING WITH STUDENT IDEAS.

Unlike the IRE sequences described in the previous section, the teaching routine described in this section has as its function the elicitation and elaboration of student ideas. The full routine is outlined as a flow chart in Fig. 1. The discussion that follows provides a brief "tour" of the flow chart.

Each of the rectangles in Fig. 1, labeled [A1] through [A7], represents a possible action by the teacher. Each of the diamonds, labeled [D1] through [D5], represents a point at which the teacher makes a decision.

In broadest outline, the routine operates as follows. In [A1], the teacher introduces a topic to the class. In [A2], the teacher invites comment and calls

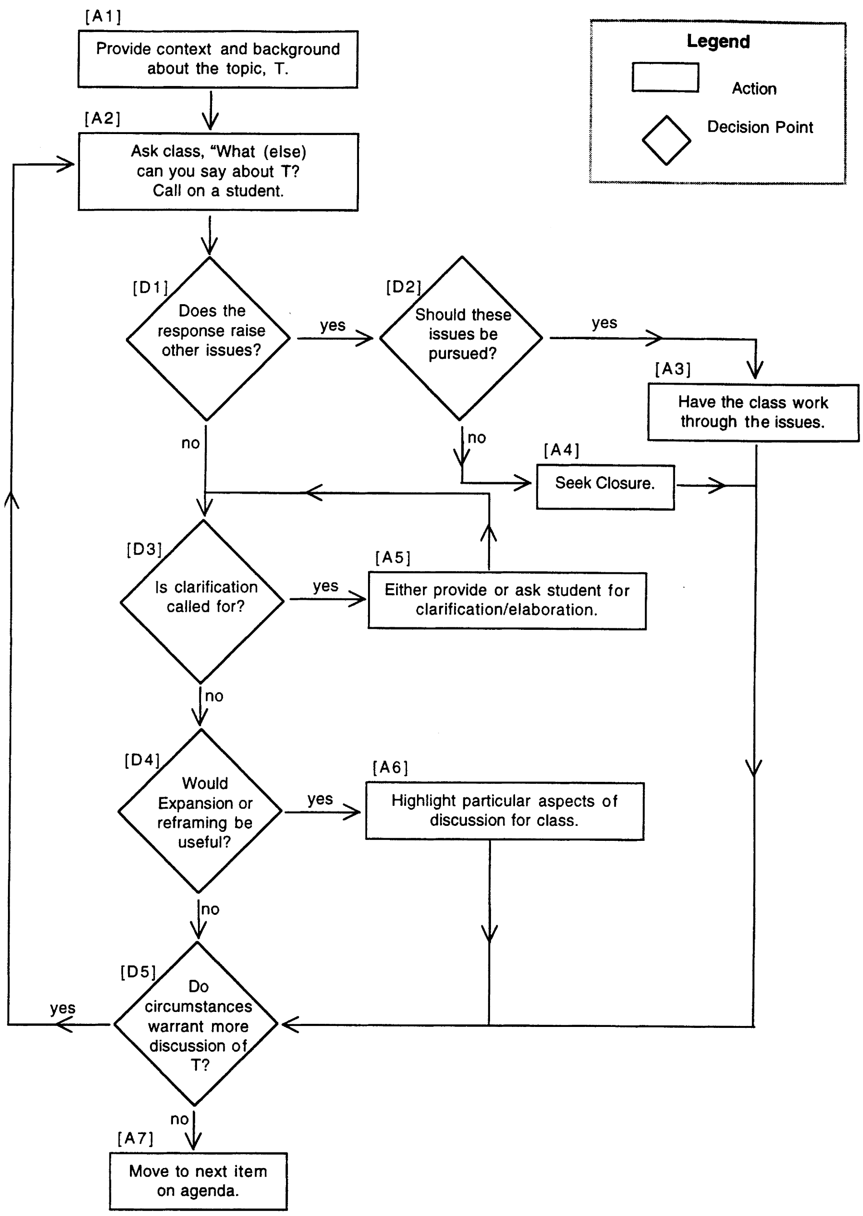


Fig. 1. A Highly Interactive Routine for Discussing a Topic.

on a student. There is always the possibility that the student's response will raise issues beyond those intended by the teacher. If it does ([D1] = "yes"), the teacher must decide whether or not to deal with those issues. That decision is represented by the right-hand branch leading from [D1]. If the student's comment is directly responsive to the teacher's prompt ([D1] = "no"), then the teacher uses that response as grist for the classroom conversation. First, the teacher decides (at [D3]) whether the class would profit from the clarification of the students' comment. If so, the teacher may prompt the student to say more, or the teacher may elaborate on what the student has said. Ultimately the student's comment is clarified to the degree deemed appropriate by the teacher. The next issue faced by the teacher (at [D4]) is whether it would be useful to expand on the student's comment, bringing particular aspects of the discussion to the class's attention. Having made that decision and acted accordingly, the teacher then decides (at [D5]) whether circumstances warrant a continuation of the discussion. If so, the teacher invites further comment. If not, the teacher makes a transition to the next item on his or her agenda.

It should be stressed that all of the teacher's decisions are highly context-dependent. Whether or not the teacher decides to ask a student to elaborate on a given point may, for example, depend on: the time left in that day's lesson; the teacher's perception of the student's readiness and willingness to pursue the idea; whether the class seems engaged; or many other factors.

In the following two sections of this chapter, I show how extended segments of dialogue in Minstrell and Ball's classrooms correspond to the routine described in Fig. 1. Two preliminary comments are necessary. First, I make no claim that Ball or Minstrell either consciously or unconsciously employed the decision procedure outlined in Fig. 1. Rather, the claim is that the routine "captures" the discourse patterns employed by the teachers – and that (see the final section of this chapter) this kind of routine may be a useful pedagogical device for teachers who wish to have their classrooms function as specific kinds of discourse communities. Second, the focus and length of this chapter preclude a detailed, line-by-line analysis of how and why these teachers made the choices they did. Detailed analyses of Minstrell's and Ball's lessons may be found respectively in Schoenfeld, Minstrell and van Zee (1999) and in Schoenfeld (1999).

ASPECTS OF JIM MINSTRELL'S "BENCHMARK" LESSON

Appendix A provides an extended excerpt (roughly 20 minutes of class time) of a lesson taught by Jim Minstrell. Here is the relevant context.

The lesson discussed here is part of a series of lessons specially designed by Minstrell as an introduction to his high school physics course. It takes place the fourth day of the course. The first two days of the course are devoted to introductory activities such as an extensive "name game" and a diagnostic test that documents the students' initial knowledge. On the third day Minstrell begins the substantive content of the course with a non-standard problem of his own design, the Blood Alcohol Content (BAC) problem. In essence, the problem is as follows. Suppose someone has been stopped for drunk driving, and five measurements of that person's blood alcohol content have been taken. You have the five numbers. Which of those numbers should be combined, in what way, to give the "best value" for the person's blood alcohol content?

The Blood Alcohol Content problem is a carefully chosen mechanism for introducing the content and social dynamics of the course. Minstrell has a number of high level goals for his students. He wants them to see physics as a sense-making activity – a way of making reasoned judgments about physical phenomena. He wants the students to see themselves as competent reasoners who are capable of sorting through complex issues themselves. He has, thus, chosen a problem that is meaningful to the students, and which they can engage fully. His discourse style will foster students' growth and autonomy: rather than evaluate student comments and questions, he will consistently (by means of an interactive technique he calls "reflective tosses") turn questions back to the students. Minstrell works to foster a classroom environment in which students feel enfranchised – an environment in which they feel it is their right (indeed, their responsibility) to raise issues and think through them carefully. Van Zee and Minstrell describe the context for the fourth lesson as follows.

> The students worked on the Blood Alcohol Content problem in small groups during the 3rd day of class. In addition, a student from each group independently measured the length and width of the same table. The numbers obtained for the width in centimeters were 106.8, 107.0, 107.0, 107.5, 107.0, 107.0, 106.5, and 106.0. Near the close of the 3rd day of class, Minstrell brought the students together for a brief discussion of reasons for using only some or using all of the numbers in the Blood Alcohol Count problem. For homework, the students were find the best value for the blood alcohol count and to decide whether the driver was drunk. They were also to calculate best values and uncertainties for the length and width of the table. Minstrell and students examined these issues on the fourth day of class during the discussion analyzed here. Minstrell described this as an 'elaboration benchmark discussion' in which he planned to work through a series of issues which the students had already opened and considered in small groups in class and on their own at home (van Zee & Minstrell, 1997, p. 240).

The first part of this fourth lesson is devoted to "housekeeping" issues related to course administration. When those issues have been dealt with, Minstrell turns to a discussion of the Blood Alcohol Content problem. Appendix A picks up the transcript of the lesson at this point. The discussion of Appendix A that follows will indicate that the flow of classroom discourse corresponds, with great fidelity, to the routine described in Fig. 1.[1]

First Implementation of the Routine: Lines 1–70

I claim that the classroom dialogue captured in lines 1–70 of the transcript can be represented by three "passes" through the routine, in lines 1–33, 34–45, and 46–70 respectively.

First Pass: Lines 1–33

Minstrell provides context and background for the discussion (step [A1] of the routine) in lines 1–12 of the transcript. He follows this in lines 13–14 by a request for student input (step [A2]). S1's response in line 15 is on target. Hence [D1] = "no," and he moves to [D3]. S1's comment in line 15 does call for elaboration ([D3] = "yes"), and Minstrell pursues the elaboration in lines 16–33. At this point neither abstraction nor re-framing is necessary ([D4] = "no"); the delineation of various contexts in which the highest and lowest values might be eliminated is sufficient. This completes the first pass through the routine. As the discussion has just begun, circumstances clearly warrant a continuation of the discussion ([D50] = "yes"). Hence Minstrell asks the students for additional comments.

This first pass through the routine is represented schematically in Fig. 2.

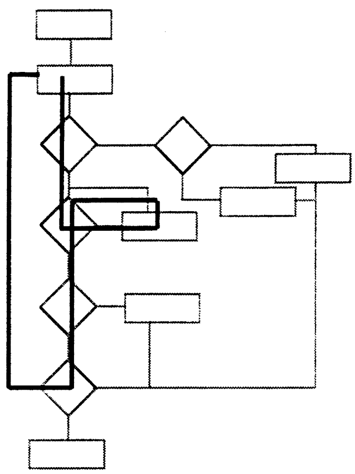


Fig. 2. A Schematic Representation of Lines 1–33 of Appendix A.

Second Pass: Lines 34–45

Minstrell begins the second round of discussion (step [A2]) in line 34. S4's response in line 36, dealing with the elimination of values that are too large or small, is on target ([D1] = "no") and does not need clarification ([D3] = "no"). Following the student's comment, Minstrell chooses to introduce a new vocabulary term ("outliers") and to expand upon the rationale for eliminating outliers ([D4] = "yes"). This completes the second pass through the routine. As there is still much to be said ([D5] = "yes"), Minstrell asks for additional input.

This second pass is represented schematically in Fig. 3.

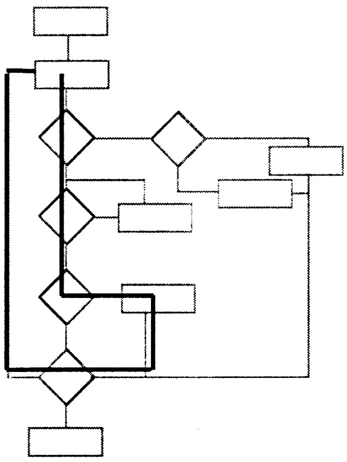


Fig. 3. A Schematic Representation of Lines 34–45 of Appendix A.

Third Pass: Lines 46–66

Minstrell asks for "another one" (step [A2]) in line 46. S4's response (lines 47–49) is again on target ([D1] = "no") and does not need clarification ([D3] = "no"). As in the previous pass, Minstrell chooses to expand upon the student's answer ([D4] = "yes"); in doing so he completes the pass. In lines 65–66 Minstrell provides the opportunity for a continuation of the discussion. When it appears that the well has run dry, he moves (lines 67–70) to the next item on the agenda (step [A8]), the issue of how best to combine the numbers.

This third pass is represented schematically in Fig. 4.

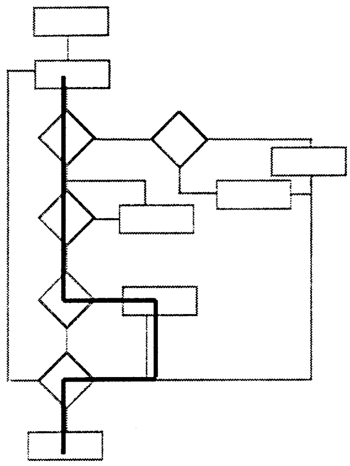


Fig. 4. A Schematic Representation of Lines 46–70 of Appendix A.

Second Implementation of the Routine: Lines 68–251

At this point in the lesson, the issue is how to best combine the given data. There are, of course, three classical measures of central tendency: mean, median, and mode. Rather than lay these out, Minstrell will ask the class "What the heck are we going to do with these numbers?" He has reason to expect, of course, that the students will generate the three measures of central tendency – and if they fail to generate one, he can always "seed" the conversation with reference to it. This situation is ideal for the use of the routine, in that the order in which the students generate ideas doesn't matter. Hence he can solicit suggestions and take them as they come.

I claim that lines 68–220 of the transcript can be represented by four passes through the routine (lines 68–89, 90–109, 110–214, and 214–220). Lines 221–251 represent the adaptive move suggested above, adding an approach to the list when the students fail to generate it themselves.

First Pass: Lines 68–89.

Minstrell begins in lines 68–70 by framing the problem of "best value" for class discussion (step [A1]), and continues in lines 71–72 by asking, "What's one thing we might do with the numbers?" (step [A2]). S5's response in line 73 is on the mark ([D1] = "no") and calls for clarification ([D3] = "yes"), which

Minstrell requests in lines 74–75. S5's definition in lines 76–77 is correct ([D3] = "no"). Minstrell decides ([D4] = "yes") to expand upon the definition in lines 78–89. This is just the beginning of the discussion, so ([D5] = "yes") he will pursue the discussion. This first pass is represented schematically in Fig. 5.

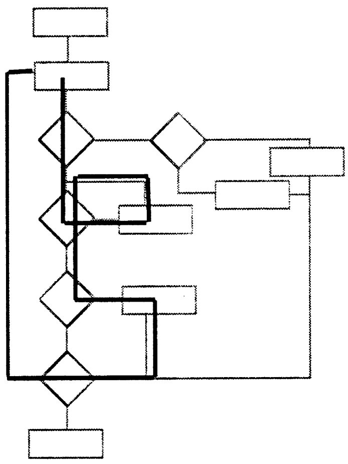


Fig. 5. A Schematic Representation of Lines 68–89 of Appendix A.

Second Pass: Lines 90–109.
Minstrell begins the second pass (lines 90–95, step [A2]) by asking if the students have "any other ideas" for computing the best value. S7's comment in line 96 begs for clarification ([D3] = "yes"), which emerges in dialogue in lines 97–104. Minstrell provides the formal definition of the term they have been discussing ([D4] = "yes") in lines 104–105 and ([D5] = "yes") moves to continue the discussion in lines 105–109.

This second pass is represented schematically in Fig. 6.

Third Pass: Lines 110–213.
Minstrell begins the third pass in line 110, with another request (step [A2]) for "another way of giving a best value." S8's response, which is non-standard, raises a number of very interesting issues ([D1] = "yes") which Minstrell pursues ([D2] = "yes") for quite some time.

A detailed examination of Minstrell's decision to follow up on S8's comments, and the way in which he did so, is fundamental to understanding how Minstrell's teaching reflects his top-level goals for his students (specifically, his goal of

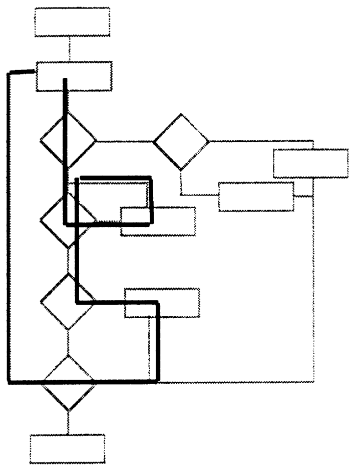


Fig. 6. A Schematic Representation of Lines 90–190 of Appendix A.

creating a discourse community that respects and encourages student initiative). That analysis, which is outside the scope of this chapter, may be found in Schoenfeld, Minstrell, and van Zee (1999). Suffice it to say here that Minstrell takes a substantial amount of time to explore the ramifications of the student's definition. In the process he covers some important subject matter and sends the message that a thoughtful suggestion from a student is important enough to warrant the expenditure of a significant amount of class time.

Minstrell wraps up this discussion in line 213 and ([D5] = "yes") invites the students to make additional suggestions.

This third pass is represented schematically in Fig. 7.

Fourth (Brief) Pass: Lines 214–220

At this point the mean and the mode have been discussed, but the median has yet to be mentioned. Minstrell begins the fourth pass in line 214, with a request (step [A2]) for another way to approach the problem. S12's response, "you could possibly take the number that appears most often," reintroduces the mode. Minstrell notes this, and then moves to bring closure to the discussion.

Coda: Introducing a Missing Element, Lines 221–251.

Due to the extended unplanned conversation in lines 110–213, it is much later in the class period than it typically would be at this point of the discussion.

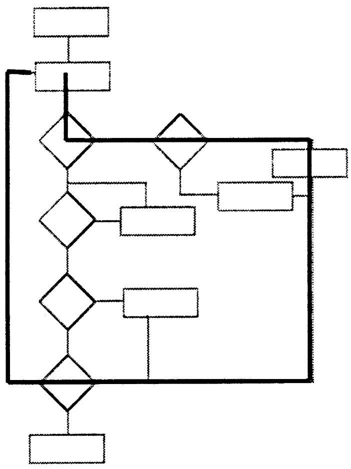


Fig. 7. A Schematic Representation of Lines 110–213 of Appendix A.

The class has generated two of the three measures of central tendency (mean and mode) but failed to generate the third (median) in response to Minstrell's invitation in line 214. Minstrell introduces it himself in line 221. This adaptive modification of the routine will be considered in the concluding discussion of this chapter.

ASPECTS OF DEBORAH BALL'S "SHEA NUMBERS" LESSON

Appendix B provides an excerpt of the first part (roughly six minutes of class time) of a third-grade lesson taught by Deborah Ball. Here is the relevant context.

This class takes place in January, mid-way through the school year. The discourse community is well established. Ball has worked with her third graders to establish a community that operates according to specific sociomathematical norms, using a vocabulary tailored to those norms. Students make *conjectures*, and they are expected to provide evidence in favor of those conjectures. When a student *disagrees* with another student's conjecture, he or she must provide reason for the dissent: "I disagree because . . ." When a student wants to retract or alter a previously expressed opinion, he or she says "*I revise my thinking*."

Ball's class has been exploring the properties of even and odd numbers. On the basis of empirical observations they have made some conjectures, for example that the sum of two odd numbers will always be even. They have also dealt with some conundrums, such as the classification of zero. All of the other whole numbers are either even or odd. Is zero even, odd, or perhaps "special"?

Part of Ball's agenda is to have the students reflect on their learning, and on the processes by which they come to understand mathematics. She wants them to understand that it takes a long time to make sense of some things – for example, that last year's third graders, now in the fourth grade, are still grappling with some of the issues that this year's class is working through. Ball had arranged for a meeting between this year's and last year's classes, to discuss even and odd numbers. That meeting took place on the day before the lesson in question. Her agenda as she opens this lesson is to "debrief" the students about their impressions of the previous day's meeting. What issues did it raise for them? She announces "I'd just like to hear some comments about what you thought about the meeting, what you noticed about the meeting, what you learned at the meeting."

As will be seen, the conversation takes some interesting twists and turns; it seems very loosely structured at first. Yet, the flow of dialogue corresponds closely to the routine discussed above: lines 1–8, 9–20C, 20D–24, 25–58, and 59–67 will be seen to correspond to five passes through the flow chart given in Fig. 1. There is much more to the analysis than can be discussed here; see Schoenfeld (1999) for details. The summary given here is derived from that analysis.

First Pass: Lines 1–8.

Ball begins the lesson in line 1 by establishing the context for the discussion (step [A1]) and (step [A2]) calling on Shekira to comment on the previous day's meeting. Shekira's comment in line 2 is on target ([D1] = "no") but needs clarification ([D3] = "yes"). Ball prompts for greater specificity (step [A5]) in line 3 and again in line 5. Given Ball's reflective agenda, Shekira's comment in line 6 does call for reframing ([D4] = "yes"). Ball does so in line 7 and ([D5] = "yes") calls on Shea to continue the discussion.

This first pass through the routine is represented schematically in Fig. 8.

Second Pass: Lines 9–20C

Events move differently in lines 9–20. Ball begins (step [A2]) by asking for more comments about the meeting. Shea's comment is not focused on the prior day's meeting, however. Rather, Shea disagrees with Shekira about an issue of mathematical content ([D1] = "yes"). Ball decides that this issue should be

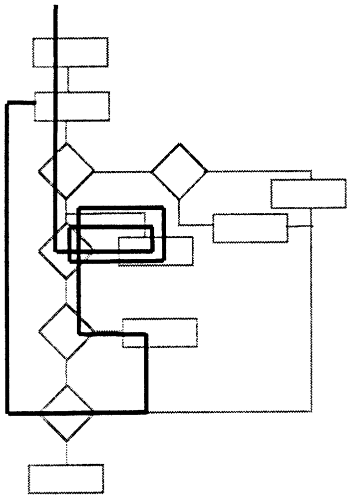


Fig. 8. A Schematic Representation of Lines 1–8 of Appendix B.

worked through ([D2] = "yes") and she and the class watch (step [A3]) as Shea and Shekira come to an uneasy accord. In line with her reflective agenda, Ball comments in lines 20A-C about how difficult some of these issues are. Then she moves ([D5] = "yes") in lines 20D–F to continue the discussion.

This pass through the routine is represented in Fig. 9.

Third Pass: Lines 20D–24

Ball invites further comment (step [A2]) in lines 20D-F, calling on Lin in line 20G. Lin's comment is on target ([D1] = "no") but invites a follow-up question ([D3] = "yes"), which Ball asks in line 22. Lin's response in line 23 stands on its own ([D4] = "no"). Still interested in pursuing her agenda ([D5] = "yes"), Ball asks for more comments. See Fig. 10.

Fourth and Fifth Passes: Lines 25–58 and 59–67

Lines 25–58 are extremely interesting – the question being why Ball, in line 26, embarked on an explicit, announced detour from her reflective agenda. A great deal can be said about this decision; see Schoenfeld (1999) for detail. That issue is beyond the scope of the current discussion. Here I restrict my attention to the routine described in Fig. 1. Ball calls for more comments (step [A2]) in line 24. From her perspective Benny's response in line 25 raises issues that she wanted to address ([D2] = "yes"). Ball works through those issues (step

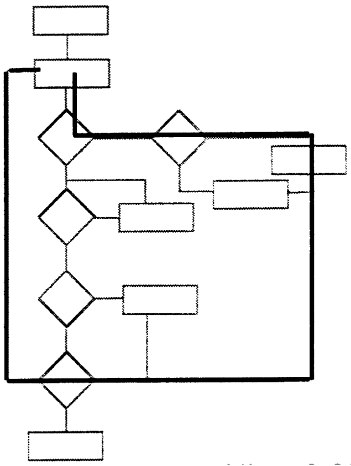


Fig. 9. A Schematic Representation of Lines 9–20C of Appendix B.

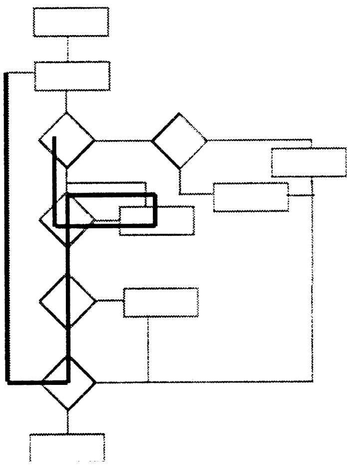


Fig. 10. A Schematic Representation of Lines 20D–24 of Appendix B.

[A3]) in lines 26C through 58. Having done so ([D2] = "yes") she returns to her reflective agenda.

In line 59, Ball starts the next pass through the routine (step [A2], "I'd really like to hear from as many people as possible what comments you had or reactions you had to being in that meeting yesterday"). Shea himself announces in line 60 that his comment is off topic. Ball misinterprets Shea's comment in lines 60 and 62 (see Ball, undated); she believes that he is addressing Benny's conjecture, which had been the focus of lines 25–58. This issue, having been resolved at some length, does not warrant further discussion ([D2] = "no"), and Ball moves to obtain closure in line 65. She still wishes to pursue her reflective agenda ([D5] = "yes"), and in line 67 she asks for more comments. These two passes through the routine are represented in Fig. 11.

DISCUSSION

If the preceding analyses are right, then the two very dissimilar-looking lesson segments taught by Jim Minstrell and Deborah Ball share, at one level of analysis, the same deep structure. Does that matter? I think it does.

This is where the discussion becomes conjectural. What follows is grounded in my reflections on the way I teach my undergraduate problem solving course

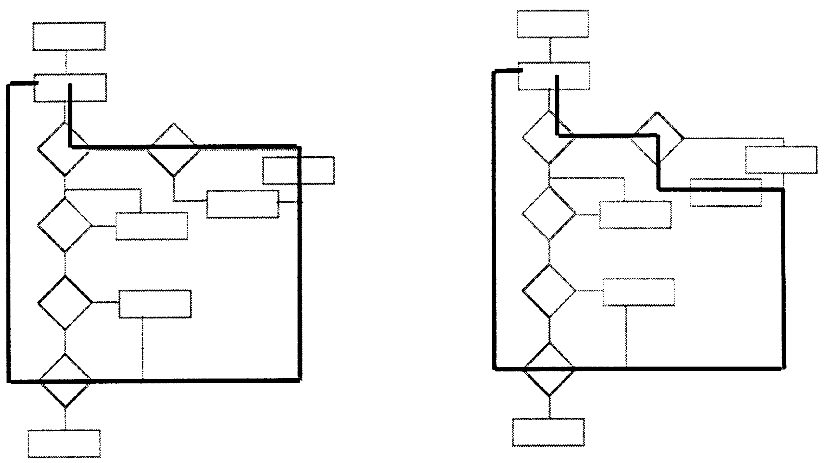


Fig. 11. Representations of Lines 25–58 and 59–67 of Appendix B, respectively.

and my understandings of Ball's and Minstrell's intentions and actions. Although the three of us are *very* different people (and teachers), we do have some goals and practices in common. The goals include creating learning environments in which our students experience mathematics or physics as a form of sense-making; in which the students reflect on their learning; and in which they develop certain productive habits of mind. The practices include the routine that has been the focus of this chapter. I suspect that the routine described in Fig. 1 – which is quite flexible, depending on the constraints the teacher imposes on it – can and often does play a significant role in the establishment and maintenance of highly interactive classroom discourse communities.

To establish the context for the discussion that follows, let me describe an apparent paradox. One of the most important goals of my problem solving course is the shaping of the classroom environment in a very particular way – as a community of independent thinkers engaged collaboratively in reasoned discourse. I have written about this goal as follows:

> The activities in our mathematics classrooms can and must reflect and foster the understandings that we want students to develop with and about mathematics. That is: if we believe that doing mathematics is an act of sense-making; if we believe that mathematics is often a hands on, empirical activity; if we believe that mathematical communication is important; if we believe that the mathematical community grapples with serious mathematical problems collaboratively, making tentative explanations of these phenomena, and then cycling back through those explanations (including definitions and postulates); if we believe that learning mathematics is empowering, and that there is a mathematical way of thinking that has value and power, then our classroom practices must reflect these beliefs. Hence we must work to construct learning environments in which student actively engage in the science of mathematical sense-making (Schoenfeld, 1994, pp. 60–61).

If I am true to my word, then my problem solving class should have a pretty free-wheeling atmosphere. Yet, when Arcavi, Kessel, Meira, and Smith (1998) analyzed my problem solving course, they found that this was decidedly not the case as the course got under way. In the first few days of the class, analysis revealed, I exercised a subtle but firm controlling hand. Is this hypocritical, or inconsistent with my avowed goals?

In a word, no. At the beginning of the semester the class had not yet developed the norms of respectful and substantive exchange that are necessary for the successful functioning of a free-wheeling community. One of my major jobs as the semester began was to encourage the students to take risks and express their opinions – but in such a way that they did so on solid mathematical grounds, and without stepping on each other's toes. The more practiced they became at this, the less I needed to provide structure. Ultimately, the community functioned on its own (see Schoenfeld, 1994).

With this as background, let me return to the routine described in Fig. 1. In a sense, the flow chart in Fig. 1 makes things look too straightforward, for there is a great deal of subtlety to the implementation. Teachers have a great deal of flexibility in implementing the routine, because the decision points [D1] through [D5] provide a large degree of latitude. Suppose, for example, that the teacher consistently declines to pursue issues other than those on his or her agenda (that is, [D2] is consistently "no"), and that when clarification is called for, the teacher provides it. The result is that classroom events, although highly interactive, will unfold very much according to the teacher's agenda. On the other hand, the teacher might encourage the students to pursue interesting issues when they arise ([D2] is consistently "yes"), and may consistently turn issues back to the students, pointing out that clarification is needed but leaving it to the students to provide it. When the routine is implemented in this way, the classroom agenda is essentially co-constructed and the classroom community is largely on its way to autonomous functioning. The same routine, then, can be made to function in different ways, depending on the "state of the community." It is reasonable to conjecture that a teacher might use the routine in somewhat constrained ways while the norms of the classroom community are being developed, and then in less constrained ways as the discourse community comes into its own.

I don't want to push the data too far, but this seems to be the case in the two lessons examined in this chapter. Minstrell's lesson is wonderfully enfranchising and interactive (especially compared to traditional presentations). At the same time, his use of the routine in Fig. 1 is also reasonably well constrained. On the one hand, Minstrell employs a large number of techniques that invite student participation in important ways. The simple act of waiting for as long as nine seconds after asking a question (thirteen seconds in other parts of the lesson) makes it clear that his questions are not rhetorical, but are meant to provoke student responses. The frequency of questions is astounding. More than half of Minstrell's dialogic turns involve posing serious questions to the students, and the students' responses provide much of the substance of the discussion. Some of that substance is clearly new – consider for example the exchange in lines 171 to 189. (Minstrell clarifies the suggested procedure and asks the class for its opinion of it. In the ensuing discussion all of the ideas come from the students.) And when a student makes an unexpected comment in lines 111–114, Minstrell devotes a large amount of time to exploring the idea she suggests.

Looked at from the students' perspective, this is a remarkably "open" lesson. Students are actively encouraged to participate and when they do the teacher picks up their ideas and runs with them – even if the discussion leads in unexpected directions. The ideas are worked through carefully. In exploring the properties of S8's proposed "average" (Is this the same as what we usually call average? Does it provide a good summary of the data?), Minstrell models the kind of discourse practices he expects: in this class, new ideas will be honored by being subjected to careful scrutiny! Over time, the students will perceive it to be their obligation to run with each other's ideas in similar ways.

On the other hand, Minstrell remains firmly in charge of the agenda for the class. His questions, while often turning responsibility back to the students, provide clear direction for the conversation; his comments often add substance to what a student has said. His one deviation from his planned agenda, in lines 110–213, has significant "value added." Pursuing S8's question honors student inquiry. It provides the opportunity to explore the properties of the arithmetic average and this proposed variant of it, both of which are plausible extensions of the lesson's content. And it models the process of exploring new ideas. The opportunity is serendipitous and the decision to pursue it spontaneous – but pursuing it is very much in line with the teacher's top-level agenda. Beyond this, it is worth noting that Minstrell's use of the routine to discuss the "best value" has a built-in safety valve. The students may well suggest all three standard measures of central tendency (mean, median, and mode). But if they only mention two, Minstrell can always mention the third himself.

In sum, Minstrell's use of the routine sets the students on the path to autonomy, by providing a structure that will ultimately support free-wheeling classroom discussion – and it is used in a way that is carefully scaffolded. This seems entirely appropriate for one of the first classes of the year.

In contrast, Ball's lesson takes place mid-year, at which point the relevant sociomathematical norms have been well established. (As one indication of this, we have Shea's comment to Shekira in turn 10. The comment is polite; it uses the technical term *disagree*; and his disagreement is backed up with an implicit appeal to the definition of evenness.)

Let us examine the first three passes through the routine. In this part of the lesson Ball plays much more of a facilitative rather than a directive role. In the first pass through the routine, Ball asks questions designed to help Shekira articulate her feelings about the meeting. During the second pass Ball stands aside while Shea and Shekira discuss Shekira's statement that zero "could be even." Her doing so is important, and reflects the state of the community. The conversation between Shea and Shekira is about the mathematics rather than about the meeting – in focusing on the properties of zero, it raises "other issues" than those in Ball's reflective agenda. By standing aside and giving Shea and Shekira room to pursue this conversation, Ball not only honors student initiative but, de facto, gives the students a role in the day's agenda-setting. In the third pass, Ball asks Lin the obvious question – in essence, "how are you going to deal with your current state of confusion?" – and then lets Lin's answer speak for itself.

These actions, I would argue, are entirely in line Ball's goals and with the capacity of the class to function as a productive discourse community. They are consistent with what happens later in the class session, when the agenda is again co-constructed (the class pursues a conjecture by Shea that the number six can be both even and odd) and the students, largely on their own, engage in extended and substantive mathematical discussions.

In sum, the routine outlined in Fig. 1 plays out very differently in the two lessons studied – appropriately so, given the state of each discourse community at the time the routine was implemented. It appears, on the basis of these lessons and my reflection on my own teaching, that this routine – tailored to circumstances – plays a useful role in shaping and then maintaining the productive exchange of ideas. For those of us who believe that classrooms should be homes to communities of reasoned discourse, it can be a useful tool.

CODA

When we were invited to contribute to this volume, Jere Brophy asked the authors to address six specific issues. I have dealt with a number of those issues tacitly in the body of this chapter, but in the spirit of cooperation, let me be explicit in addressing them here. The questions and my responses follow.

What Does Social Constructivist Teaching Mean in the Area(s) of Teaching on Which your Scholarly Work Concentrates?

I hate to start off on an oppositionist note, but I have some serious difficulties with the phrase "social constructivist teaching." For me, social constructivism is a theoretical perspective that can be used to help understand what happens in classrooms – any classrooms. As such, social constructivism doesn't represent or endorse a particular kind of teaching. Be that as it may . . .

I view mathematics as a particularly powerful and empowering lens through which one can make sense of the world. Mathematics coheres – it fits together, and one can make sense of it. I want students to experience mathematics this way, and to come away from their mathematics instruction with a sense of themselves as competent and autonomous reasoners. I may have gone a bit overboard rhetorically in the segment of Schoenfeld (1994) quoted above, but I still believe the bottom line: If we want students to become mathematical sense-makers, we need to construct learning environments in which they actively engage in mathematical sense-making.

Some clarification is necessary here – I want to avoid extremes. On the one hand, a steady diet of straight didactic presentations and imitative exercises deprives students of autonomy and of the sense that they are capable of doing mathematics on their own. On the other hand the equally extreme alternative, a caricature of discovery learning in which students are given interesting problems and set loose with little guidance, is also untenable. The "sink or swim" approach is no more appropriate for learning to think mathematically than it is for learning to swim.

What I think *is* appropriate is a carefully chosen combination of curriculum and pedagogy. What we know in curricular terms is that it is not necessary for students to be taught everything, and then to engage in imitative exercises: it is possible for students to learn (some) things by solving problems, rather than learning things first and then applying what they have learned to so-called "problems." We have also seen – and Ball's and Minstrell's classrooms are prime examples – that students are capable of much more sophisticated reasoning than we tend to give them credit for. However, classrooms that support that kind of reasoning do not tend to occur by spontaneous generation. It is an act of great pedagogical skill to shape a classroom discourse community so that it facilitates productive exchanges among students. It takes vigilance to maintain such a community – although, paradoxically, the "presence" of the teacher may seem diminished as the students become more autonomous and the community seems to function more "on its own." Yet such intellectual communities – classrooms in which students participate in disciplinary sense-making that is structured and scaffolded where necessary and appropriate – are what I hope to see more of.

What is the Rationale for Using These Methods, and What Forms do they Take?

There is a large body of research indicating that students develop their sense of the mathematical enterprise from their experience in mathematics classrooms. The consequences of traditional didactic instruction are all too well known (recall the quote from Lampert, 1990; see also Schoenfeld, 1992; Voigt, 1989). Increasingly, there are "existence proofs" of the kind discussed here, where students learn to engage in disciplined inquiry. Moreover, there is now compelling evidence that some of the new "reform" curricula in mathematics are producing gains on (oh yes!) standardized tests (Schoenfeld, 2001).

The rationale for the "middle ground" approach suggested above is simple. Students are much more likely to develop productive habits of mind when they have the opportunity to practice those habits, and to develop a disposition toward sense-making when they are members of communities that engage (successfully!) in such practices. As suggested above, crafting such communities takes a good deal of work. People are not born knowing how to interact respectfully and productively; they have to be taught to do so. In each classroom, the "didactical contract" needs to be established and negotiated. Students often begin a course with the default assumption that this course, like others, will be run according to the "standard rules;" in courses that operate differently, different expectations need to be made explicit. Moreover, a fair amount of scaffolding is likely to be necessary. In Schoenfeld (1994), for example, I describe the way in which I explicitly violate the normative expectation that my job as teacher is to evaluate the correctness of the arguments they propose.

> The second day of class . . . a student volunteered to present a problem solution at the board. As often happens, the student focused his attention on me rather than on the class when he wrote his argument on the board; when he finished he waited for my approval or critique. Rather than provide it, however, I responded as follows:
>
>> Don't look to me for approval, because I'm not going to provide it. I'm sure the class knows more than enough to say whether what's on the board is right. So (turning to class) what do you folks think?
>
> In this particular case the student had made a claim which another student believed to be false. Rather than adjudicate, I pushed the discussion further: How could we know which student was correct? The discussion continued for some time, until we found a point of agreement for the whole class. The discussion proceeded from there. When the class was done (and satisfied) I summed up.
>
> This problem discussion illustrated a number of important points for the students, points consistently emphasized in the weeks to come. First, I rarely *certified* results, but turned points of controversy back to the class for resolution. Second, the class was to accept little on faith. This is, "we proved it in Math 127" was not considered adequate reason to accept a statement's validity. Instead, the statement must be grounded in mathematics solidly understood by this class. Third, my role in class discussion would often be that of Doubting Thomas. That is, I often asked "Is that true? How do we know. Can you give me an example? A counterexample? A proof?", both when the students' suggestions were correct and when they were incorrect. (A fourth role was to ensure that the discussions are respectful – that it's the mathematics at stake in the conversations, not the students!)
>
>> This pattern was repeated consistently and deliberately, with effect. Late in the second week of class, a student who had just written a problem solution on the board started to turn to me for approval, and then stopped in mid-stream. She looked at me with mock resignation and said "I know, I know." She then turned to the class and said "O.K., do you guys buy it or not?" [After some discussion, they did]" (Schoenfeld, 1994, pp. 62–63. Reprinted with permission).

The net result of this kind of interaction was that by the end of the semester, the class was challenging assertions much more regularly, demanding solid rationales, and often deciding autonomously whether or not an argument that had been presented was indeed correct. I still played the role of Doubting Thomas on occasion, and I had no hesitation in weighing in when I judged that my mathematical input was needed – but in many ways my intervention was needed less than at the beginning of the course.

It is difficult to abstract this kind of interaction into a general rule (and I distrust general rules). Parenthood may not be a bad metaphor, however. The idea is to turn over as much to the students as one thinks they can handle responsibly – and to be nearby with a safety net, just in case.

What are the Strengths/Areas of Applicability of these Teaching Methods, and What are their Weaknesses/areas of Irrelevance or Limited Applicability?

Teaching for deep understanding is hard. It calls for a substantial amount of understanding and flexibility on the part of the teacher – the willingness to explore ideas as they come up, the ability to make judgments about what might be productive directions and what might not, and the ability to provide the "right" level of support for students individually and collectively. Few teachers have had the relevant kinds of experiences as students, much less as teachers. Nor do we, at present, provide opportunities for "on the job training."

As I suggested above, what is needed is a combination of particular kinds of curriculum and pedagogy; assessment plays a critical facilitating or inhibiting role as well. Various "reform-oriented" curricula developed after the issuance of the 1989 NCTM *Standards* support some of the practices discussed here. Evidence is mounting that when teachers are provided professional development consistent with those curricula, and assessments are aligned with them, that students learn a lot more (see Schoenfeld, 2001).

When, Why, and How are Social Constructivist Methods used Optimally?

I'm not sure I can address this question, partly because there's a "chicken and egg" problem. I teach a problem solving course at the college level. In many ways it's "remedial" – I have to teach some things I'd hope my students would have learned long ago. Because of this, I focus more on thinking (problem solving strategies; habits of mind) than I do on specific subject matter content. But, I can imagine a world in which students had learned to think mathematically from kindergarten on . . .

Even so, optimality is going to be elusive for quite some time – until we have much more widespread experience with teaching techniques and curricula as discussed in this chapter. Optimality will also, I think, always be a question of values. Clearly, I value a certain kind of mathematical disposition, and certain habits of mind. But, what do students absolutely have to know? Will I be content if my students can regenerate some things, rather than recalling them, or if they know where to look them up? Your answers may differ from mine. And depending on the answers you give, your pedagogical practices may vary.

When, Why, and How do these Methods Need to be Adjusted from Their Usual Form in Order to Match the Affordances and Limitations of Certain Students, Instructional Situations, etc.?

I think the discussion of the two examples in this chapter suggests an answer, though the answer may be more vague than the reader would like. In some sense, everything is context-dependent. One has certain values and certain goals for one's students; one makes certain (research-based) assumptions about the kinds of environments that will help students attain those goals. The "rest" is scaffolding. But that's easy for me to say . . . .

I'd be tempted to leave it at that, but I do have to address one pernicious misconception regarding such issues. Some will argue that the kinds of practices discussed here are OK for "bright kids," but that "slow kids" need more didactic instruction. To pursue that path is only to exacerbate inequities. Evidence is now becoming available that "reform-oriented" instruction works across the boards. Not only do more students do well when they engage meaningfully with mathematics, but fewer students "bottom out" (Briars, 2001; Briars & Resnick, 2000; Schoenfeld, 2001).

When and Why are these Methods Irrelevant or Counterproductive (and What Methods Need to be Used Instead in these Situations)?

This is a matter of belief, and a matter how extreme one is willing to be (on either the didactic or the "teaching for understanding" side). Take a simple procedure like the one for subtraction. From one perspective, there's one right way to do subtraction (the "standard" algorithm) and the most effective way to teach it is to drill students on it. From another perspective, what counts is understanding the algorithm. If you do, you'll be flexible and have a number of different ways to do subtractions.

Which is "right"? I'm reminded of a story I was told long ago by Fred Reif. Fred needed a blood test. The technician at his HMO said "please give me the index finger of your left hand." Fred said, "I play the viola and I have a rehearsal tonight. Please use the right hand instead." The technician then said, "please give me the index finger of your left hand." Fred repeated his request. The technician thought long and hard, and then said "I guess that would be OK."

On the one hand, we can all be horrified at the thought of a technician in an HMO who doesn't know whether it's OK to take blood from the right hand instead of the left. On the other hand, Fred pointed out that there are costs involved. Would you want a doctor to be doing all the blood tests? That could get expensive. The odds are that less than one patient in a hundred causes the kind of problem Fred did, so the limited training the technician received was adequate the vast majority of the time. The optimal solution lies somewhere between those two extremes – and what you decide is optimal depends on your values.

NOTE

1. I am not claiming that Minstrell, consciously or unconsciously, follows this routine in an explicit way – any more than one would claim traditional teachers consciously employ IRE sequences. The claim, rather, is that the routine in Fig. 1 serves as a remarkably accurate post hoc description of the discourse patterns in Minstrell's classroom.

ACKNOWLEDGMENTS

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REFERENCES

Arcavi, A., Kessel, C., Meira, L., & Smith, J. P. (1998). Teaching mathematical problem solving: An analysis of an emergent classroom community. In: A. Schoenfeld, J. Kaput & E. Dubinsky (Eds), *Research in Collegiate Mathematics Education*, *III* (pp. 1–70). Washington, D.C.: Conference Board of the Mathematical Sciences.

Ball, D. L. (Undated). Annotated transcript of segments of Deborah Ball's January 19, 1990 class. Distributed by Ball at the research pre-session to the 1997 annual NCTM meeting, San Diego.

Briars, D., & Resnick, L. (2000). Standards, assessments – and what else? The essential elements of standards-based school improvement. Manuscript submitted for publication.

Briars, D. (2001). Mathematics Performance in the Pittsburgh public schools. Presentation at a Mathematics Assessment Resource Service conference on tools for systemic improvement, San Diego, CA.

Cazden, C. (1986). Classroom discourse. In: M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (3rd ed., pp. 432–463). New York: Macmillan.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, *27*(1) 29–63.

Leinhardt, G. (1993). On teaching. In: R. Glaser (Ed.), *Advances in Instructional Psychology* (Vol. 4, pp. 1–54). Hillsdale, NJ: Erlbaum.

Leinhardt, G., Weidman, C., & Hammond, K. (1987). Introduction and integration of classroom routines by expert teachers. *Curriculum Inquiry*, *17*(2), 135–176.

Mehan, H. (1979). *Learning lessons.* Cambridge: Harvard University Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: MacMillan.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In: A. Schoenfeld (Ed.), *Mathematical Thinking and Problem Solving* (pp. 53–70). Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, *4*(1), 1–94.

Schoenfeld, A. H. (1999). *Dilemmas/decisions: Can we model teachers' on-line decision-making?* Paper presented at the Annual Meeting of the American Educational Research Association, Montreal, Quebec, Canada, April 19–23, 1999.

Schoenfeld, A. H. (Ed.) (1999). *Examining the complexity of teaching.* Special issue of the *Journal of Mathematical Behavior*, *18*(3).

Schoenfeld, A. H. (2001). Making mathematics work for our kids: Issues of standards, testing, and equity. Manuscript submitted for publication.

Schoenfeld, A. H., Minstrell, J., & van Zee, E. (1999). The detailed analysis of an established teacher carrying out a non-traditional lesson. *Journal of Mathematical Behavior*, *18*(3), 281–325.

Sinclair, J., & Coulthard, R. (1975). *Towards an analysis of discourse: The English used by teachers and pupils.* London: Oxford University Press.

U.S. Department of Education (1997). Attaining excellence: TIMSS as a starting point to examine teaching. Eighth grade mathematics lessons: Unites States, Japan, and Germany. Videotape ORAD 97-1023R. Washington, D.C.: Office of Educational Research and Improvement.

Voigt, J. (1989). Social functions of routines and consequences for subject matter learning. *International Journal of Educational Research*, *13*(6), 647–656.

van Zee, E., & Minstrell, J. (1997). Using questioning to guide student thinking. *Journal of the Learning Sciences*, *6*(2), 227–269.

Zimmerlin, D., & Nelson, M. (1999). The detailed analysis of a beginning teacher carrying out a traditional lesson. *Journal of Mathematical Behavior*, *18*(3), 263–280.

APPENDIX A: EXTENDED SEGMENT OF JIM MINSTRELL'S CLASS DISCUSSION OF MEASUREMENT

T: OK. So we've talked a bit about Blood Alcohol Count, and then we've also got the table measurement to include in this. So those are the contexts in which we can talk about measurement. And let me see if I can remember where we were on Friday in terms of the discussion. And let's see.
You can help me out, ah, but I think one of the topics that we had talked a bit about was getting, what I ended up calling, a best value. Getting the best number we can. What is the blood alcohol count number for this person? What is the number for the length of that table? or for the width of that table?
And on Friday, some people said, 'let's take all the numbers' and some said, 'let's only take some numbers.' So that was one of the issues that we came up with, whether to take all the numbers that were used in the measurement or whether to only take some of the numbers.
And some of the reasons that we listed for taking some numbers were – what was one of them?
Sl: Eliminate highest and lowest.
T: OK. You might want to eliminate highest and lowest. [writes on board] Is there a context in which, ah, that's done? A measurement context that you can think of where that's done?
S2: Math
T: In some, in some math situations?
S?: [unintelligible]
T: Pardon me?
S?: [unintelligible]
T: You always do that.
S?: [unintelligible]
T: Teachers always do that, do they? Teachers always eliminate the highest and lowest?
Ss: [overlapping unintelligible student comments]
T: OK. Sometimes, ah, sometimes in some classes the teacher will, ah, or students will, or teachers will allow students to, or whatever it is, to eliminate the highest and lowest.
S3: [partially intelligible]: sometimes scores are eliminated in diving.
T: OK. So in diving sometimes you eliminate the highest and lowest? Often in the measurement of diving, you have all those judges eliminate the top and bottom number and take the rest and do something with the rest of them.
OK.What's another way at going at taking some of the numbers and not all of them? [S4]?
S4: The ones that [unintelligible comment involving eliminating extreme values]

T: OK. So we might eliminate – You need to let me know if I'm writing too small or if
you don't understand the words that I'm writing down. O.K.?
T: That's 'eliminate.' [points to abbreviation on board] All right. Eliminate, ah, what I'll
call an outlier. If there are numbers that are just completely out of the ballpark, I mean
the rest of these are sort of in a ballpark in there and then there's one that is just way
out of there, or two, that are just way out of there, or something like that, then you
might just eliminate the outliers, possibly, O.K.?
T: And keep all the measures that seem like they're pretty much in the same ballpark.
That's another, ah, decision outcome that you might come to.
O.K.? What was another one?
S4: What about, [it said like in the law that you, um, had like a certified nurse and a doctor
I think it was, and you have to eliminate certain people who weren't certified to test the
blood or to take the blood].
T: O.K. Can you hear her back there [S5]?
S5: No.
T: No? All right. You want to say that a little louder?
S4: You have to eliminate the ones that weren't absolutely certified to take the blood test.
T: O.K. In that context or essentially in general then you might want to take those
numbers, ah, done by quote the experts, something like that, and then, ah, in there
there was a special context like, ah, like for example, ah, some people said, "Oh,
goodness, the MD is the one who should do it" and others disagreed with that as
being, that person as being the expert; some people said, "Oh what's important is that I
take MINE, my measurement, because my measurement I KNOW is right, but
anybody else's, I don't know, but MINE I know is right." So there are several ways of
getting at 'experts' there; you might even want to question, you know, Who is the
expert?" Am I really an expert here?" or "Is the MD really an expert there? so there's
some question in there when you start taking it from the experts, you know – who's to
decide who the experts are?
O.K.? Any other reasons you can think of to only take some of the numbers?
[9s pause]
O.K. I think that's pretty much the list we had on Friday.
All right. Now. We're trying to get a best value and we might take all of the numbers
or we might take some of the numbers and then it's, what the heck are we going to do
with those numbers? O.K.?
So now we've got some numbers there, what are we going to do with those numbers?
What's one thing that we might do with the numbers? [S5]
S5: Average them.
T: O.K. [writes 'average them' on board] We might average them. Now what do you
mean by 'average' here. [S5]?
S5: Add up all the numbers and then divide by whatever amount of numbers you added
up.
T: All right. That is a definition for "average".
In fact, that's what we'll call an "operational definition". An operational definition is a
definition where you, where you give a recipe for how to find what it is that you're
talking about. And in this particular case, she's saying, "Add the number of – whether
you're talking about some of the numbers or all of the numbers – add those up and
divide by however many there are. And that's called the arithmetical average and to get
that you add them up and divide by how many there are. O.K.?

[talking while writing this on the board] That's an average that you often use in lots of different contexts and it's an average that we'll use in here, but look out! because there are lots of times when that's not the best average to use. On finding the best value, that's a pretty good way to get the arithmetic, or the arithmetic average is a pretty good way of getting a best value. O.K.?

Any other suggestions for what we might do? So we can average them – [8s pause]

Any other suggestions there for what we might do to get a best value?

I'll put up the numbers that we had from the table measurement on Friday in first period, for the length, for the width and the length. As you look at that array of numbers, any other ideas there that come to mind as to how you might go about getting a best value from these numbers you're going to take there? [S7]?

S7: You've got a bunch of numbers that are the same number.

T: O.K. Like what are you talking about there?

S7: 107.

T: All right. 107 point zero, 107 point 0, 107 point 0, 107 point 0.

Is there any other number in the width column that shows up as much as 107 point zero?

S: No.

T: No. OK? So it's the number that shows up the most often is another way of picking one. That's called the, um, the mode. OK? [writes on board] The number that shows up most frequently. OK. That's another way of getting a best value out of a collection of numbers that you're willing to keep.

Does that make sense? Anybody confused here? yet? Haven't confused anybody yet? Then I've got to push a little harder. [4s pause]

All right?

Anybody think of another way of giving a best value? [S5]?

S8: This is a little complicated but I mean it might work. If you see that 107 shows up 4 times, you give it a coefficient of 4, and then 107.5 only shows up one time, you give it a coefficient of one, you add all those up and then you divide by the number of coefficients you have.

T: You lost me.

S?: [unintelligible] [overlapping student comments]

S8: One of those numbers. It's just that the more times it shows up, that makes like makes it a more, um, a more weight.

T: OK. Let me see if I can follow what you're saying then. You're saying one zero seven point zero shows up four times [writing on board] so let me put a multiplier in front of it, {sotto voce: that's what a coefficient is}, of four, and then what, what am I going to do?

S8: Ah, you average that, well then you, just say there are, ah, five numbers, and another one is.

T: Well let's go ahead and use this first column right here.

S8: OK. Then, ah, well, [unintelligible]

T: So everything else only comes up once.

S8: Wait. One, yeah, looks like it. So everything else just gets one.

T: All right. So one and, ah, we've got, ah, one oh six point eight.

S8: Eight.

T: And one?

S: Oh seven point five.

S8: Oh seven point five.
T: One oh seven point five. And one?
S?: Six point five
T: One oh six point five.
S?: One oh six.
T: One, one of six. O.K. Now what do I do?
S8: You add all that.
T: O.K.
S8: And you divide by, [muttering], eight.
T: One, two, three, four, and four makes eight?
O.K.?
[Instructor has written on the board:

$$\frac{4\ (107.0) + 1(106.8) + 1(107.5) + 1(106.5) + 1(106.0)}{8}$$

T: All right. What do you think of that method?
Ss: [overlapping student comments including "Forget it." "Too hard."]
T: Too hard?
Ss: [overlapping unintelligible student comments including "It's the same"]
T: All right. So actually it ends up being the same as the arithmetic average?
S8: No. Because 107 gets four times the value, so the 107 counts more.
T: Ah. O.K. If you were to take the arithmetic average of these numbers, what would you do? What would be the operations that you would go through there? [S5], you were the one who suggested arithmetic average.
S5: You'd add all the numbers together and then divide it by 8.
T: Now what do you mean by 'adding all the numbers'?
S5: You would add each separate number that everybody got; you wouldn't just add one 107, you'd add all the 107s.
T: O.K. All right. So what [S5] is suggesting is for an arithmetic average is to add this number, then add this number, then add this number, even though it's a repeat of that one, then add this one, this one, and this one?
Ss: [overlapping unintelligible student comments]
T: Now would that come out the same as this if you did this?
Ss: [overlapping Yeah, yes, it would]
T: All right. So if you just took the arithmetic average by adding each one of these numbers, all eight numbers, and divide it by eight then, that would end up giving you the same number as this, so this is kind of maybe a quickie way of grabbing some of them, but outside of that, it it gives us the same answer?
S8: Yeah. It does. I didn't mean it to when I did it though.
T: OK. What about this other method that, ah, that was mentioned, of saying, let's just add up the numbers that are different? like 106.8 and 107.0, 107.5, 106.5 and 106.0, that's all our different numbers, right?
S9: Why 107.0?
T: Well, because that's a, that's a different.
S10: It's different.
T: I mean, there's at least one of those, at least one of these, at least these, etcetera, add those up and then take that and divide by 5. How do you like that?
S?: No.
S?: That doesn't show, doesn't represent it truthfully though, 'cause, I mean, there's a lot

more 107s and that's be, that'd change.
T: O.K. Wouldn't that give us the same answer as if we just took the arithmetic average?
Ss: [overlapping "no"s]
T: Can you give me an instance that's a real clear example, that would drive home to me as to why that would give me a different number than if I took all of these and divided by eight? We can do that with the numbers and see that it would come out different. [S11]?
S11: If everybody got 107 except for one person who got 99 and then if you took 107 and 99 and divided it by 2, it'd be a lot different.
T: Does that make sense?
Ss: [overlapping agreement]
T: So if people go back there and measure the table, 107, 107, 107, 107, 107, 107, 107, 107, 107, 107, 107 and somebody else gets 99, so we go over there and we say, Hmm, ah, half way between 107 and 99.
Ss: [unintelligible comments]
T: Does that make sense?
Ss: [overlapping comments, "no"]
T: Now I want you to listen to yourselves because a lot of you are saying, "that's a ridiculous situation; of course, it wouldn't be half" – what is half way between 99 and 107?
S?: 103
T: 103?
S?: Yeah.
T: O.K. Of course it couldn't be 103, right? But you know what? There are going to be some contexts within here in which some of you are going to fall into that very trap right there, if you're not careful. O.K.? So watch out, so watch out for it. Is it clear that one oh, what'd I'd say, 103 would not be a good average for 99 and then all those 107s? Is that clear? O.K. All right. Ah. O.K. So this is really not a very good way to do it. Do we agree there? Somehow we need to weight, to weight in there the fact that 107 occurs so many times. So we've got this way of doing it, or if we added them all up in there, that would include all those 107s. O.K.? [4s pause]
Anybody confused yet? [2s pause] No? [2s pause]
O.K. [3s pause] Got to be honest. [4s pause]
All right. Anybody else see a different way of approaching?
S12: You could possibly take the number that appears most often, like you were saying before, if everyone got 107, and then a couple of people got 99, or like one person got 99 and one person got 120, you could pretty much assume that 107 would be nearest to the correct answer and [so that you could just select that].
T: O.K. And that's the one that we called the mode there; it shows up the most frequently. O.K. The mode.
There's another measure in here that, ah, that, ah, is sometimes used and nobody mentioned it but I'll, I'll go ahead and throw it in here then; it's what's called the 'median' measure. Anybody know what the median is?
S?: Half way.
S?: Half.
T: Yeah. If you were to take, if you were to take all of the numbers – this is getting pretty messy there, let me clean that up a bit – if we were to take all of these numbers and rank them, [writing on board] the highest one is 107.5, then it's 107, 107, 107, 107,

and then 106.8, 106.5, 106.0, do you see what I did there?

S?: Um hum.

T: I, what's called ranked them, from the biggest measure that we got to the smallest measure that we got for the width of that table, and then after I rank all of those, I go for the middle number, the middle number. Oh Beep, what do I do here? The middle number right in there.

S?: They're the same number so it doesn't matter. Is that zero then? 'Cause it's right in there? [pointing between two numbers]

Ss: [overlapping comments: no]

T: Nah. 107's above it, 107's below it, right in there, I might even go half way between these two if they differed maybe, but the median number in this case would probably be a nice 107. O.K.? So the median is "the [writing on board] middle number when all are ranked." And ranked, you know, like from the top to the bottom, etc., so then you take the middle number when all the numbers are ranked there. You have to rearrange all the numbers and then take the middle one. And that's called the median. That make sense?

S?: Sure.

T: OK. All right. Now those are some of the, ah, some of the ways then that we might, we first of all might take all of the numbers to get a best value, or we might take some of those numbers to get a best value, then what we might do with them is that we might average them, or we might go for the number that shows up most frequently, or we might go for the middle number. These are all different techniques for getting a best value.

APPENDIX B: DEBORAH BALL'S CLASS; FRIDAY JANUARY 19, 1990 THIRD GRADE, SPARTAN VILLAGE SCHOOL, EAST LANSING, MICHIGAN

1 Ball: **[A]** Okay. A few delays, but I think we're ready to start now.
[B] I'd like to open, open the discussion today with um – I have a few questions about the meeting yesterday that I'd like to ask.
[C] So, to begin with, I would just like everybody to put pens down, there's nothing to take notes about or do right now.
[D] But I'd like you to be thinking back to yesterday and to the meeting that we had on even and odd numbers and zero.
[E] And I have a few questions. First – my first question is, I'd just like to hear some comments about what you thought about the meeting, what you noticed about the meeting, what you learned at the meeting, just what kinds of comments you have about yesterday's meeting?
[F] And could you listen to one another's comments, so that we can um, benefit from what other people say?
[G] See what y– what you think about other people's comments? Shekira, do you want to start?

2 Shekira: I– I– I liked it because, well, I like talking to other classes and, and when you talk to other classes sometimes it helps.

3	Ball:	In what way?
4	Shekira:	It helps you to understand a little bit more.
5	Ball:	Was there an example of something yesterday that you understood a little bit more during the meeting?
6	Shekira:	Well, I didn't think that zero was – zero, um – even or odd until yesterday they said that it could be even because of the ones on each side is odd, so that couldn't be odd. So that helped me understand it.
7	Ball:	Hmm. So y– So you thought about something that came up in the meeting that you hadn't thought about before? Okay.
8	Shekira:	*(nods)*
9	Ball:	Other people's comments? Shea?
10	Shea:	Um, I– I– I just want to say something to Shekira, when sh– what she said about um that, that one, um – zero has to be an odd, an even number bec– I disagree because, um, because what what two things can you put together to make it?
11	Shekira:	Could you repeat what you said, please?
12	Ball:	*(speaks to Bernadette and asks her to listen to this)*
13	Shea:	Okay, um, I disagree with you because, um, if it was an even number, how – what two things could make it?
14	Shekira:	Well, I could show you it. (*Moves toward the chalkboard and points to the number line above the chalkboard.*) Um, I forgot what his name was – but yesterday he said that this one (*points to the 1 on the number line*) and each – this one is odd and this one (*points to the −1 on the number line*) is odd, so this one has to be even.
15	Shea:	But, that doesn't mean it always is even.
16	Shekira:	It *could* be even.
17	Shea:	It *could* be, but . . .
18	Shekira:	I'm not saying that is *has* to be even. I meant that it could be.
19	Shea:	You said it was.
20	Ball:	**[A]** Before we take this up again, I underst– I– I understand that this is still a problem and that we didn't a – we didn't settle it, we're probably not going to settle it. **[B]** Um, there's a lot of disagreement about this issue, right? **[C]** And you saw that the fourth graders who have been thinking about this for a long time also disagree about it, don't they? **[D]** I'm still kind of interested um, in hearing some more comments about the meeting *itself*. **[E]** Shekira commented that it was good to have the two classes together because she heard an idea that she hadn't thought about and it made her think about and even revise her own idea when she was in the meeting yesterday. **[F]** What other comments do other people have about the meeting and what happened yesterday? **[G]** Lin, do you have a comment?
21	Lin:	Um, I h– I thought that zero was always going to be a even number, but from the meeting I sort of got mixed up because I heard other ideas I agree with and now I don't know which one I should agree with.
22	Ball:	Um– hm. So what are you going to do about that?
23	Lin:	Um, I'm going to listen more to the discussion and find out.

24	Ball:	Other people? Benny?
25	Benny:	Um, first I said that um, zero was even but then I guess I revised so that zero, I think, is special because um, I – um, even numbers, like they they *make* even numbers; like two, um, two makes four, and four is an even number; and four makes eight; eight is an even number; and um, like that. And, and go on like that and like one plus one and go on adding the same numbers with the same numbers. And so I, I think zero's special.
26	Ball:	**[A]** Can I ask you a question about what you just said? **[B]** And then I'll ask people for more comments about the meeting. **[C]** Were you saying that when you put even numbers together, you get another even number –
27	Benny:	Yeah.
28	Ball:	– or were you saying that all even numbers are made up of even numbers?
29	Benny:	Yes, they are. [This is very hard to make out. There has been significant dispute over whether Benny said "yes they are" or "no, they're not."]
30	Ball:	Bernadette, you said something like that yesterday, too.
31	Bernadette:	What.
32	Ball:	Were you – were you not listening to this just now?
33	Bernadette:	No.
34	Ball:	Benny said a minute ago that when you put even numbers together you get an even number,
35	Bernadette:	Mm-hm.
36	Ball:	But he also said, I think, that all even numbers are made up of other even numbers.
37	Lin:	I disagree.
38	Shekira:	(*says something to Lin*)
39	Ball:	Two even numbers just the same.
40	Benny:	Unh-uh.
41	Ball	The same even number?
42	Benny:	Yeah, like four.
43	Ball:	**[A]** Like eight is four plus four? **[B]** Are all the even numbers – can you do that with all the even numbers? That they'd be made up of two identical even numbers?
44	Shea:	Not– not– not–
45	Bernadette:	(*looking toward Benny*) You can't. Like six. Six is two, two . . . . Six you can't get two.
46	Shea:	Six is two *odd* numbers to make an even, to make an even number.
47	Lin:	Three and three –
48	Bernadette:	(*still looking toward Benny*) You need three twos to make six. You can't put a four and a four or a . . .
49	Shea:	Three twos???
50	Bernadette:	(*looking toward Benny*) Three's – Three is odd.
51	Shea:	Or, um –
52	Benny:	I know that, but um, um I'm talking about like two plus two is four, and four plus four is eight and I just skipped the six so I just added the ones that, that add. Like the two plus two is four, and four is an even number and I'm just talking about the things that um, like –
53	Shea:	Six can be an odd number.

54	Benny:	What I just said – the um, like two is plus two is four and four plus four is eight and –
55	Bernadette:	So what you're doing is you're going by twos and then what two equals from then you go from – all the way up.
56	Benny:	Yeah, I'm not going by every single number. Like,
57	Bernadette:	Okay.
58	Benny:	Two, four, six, eight.
59	Ball:	**[A]** More comments about the meeting? **[B]** I'd really like to hear from as many people as possible what comments you had or reactions you had to being in that meeting yesterday. **[C]** Shea?
60	Shea:	Um, I don't have anything about the meeting yesterday, but I was just thinking about six, that it's a . . . I'm just thinking. I'm just thinking it can be an odd number, too, 'cause there could be two, four, six, and two, three twos, that'd make six . . .
61	Ball:	Uh-huh . . .
62	Shea:	And two *threes*, that it could be an odd and an *even* number. Both. *Three* things to make it and there could be *two* things to make it.
63	Ball:	And the two things that you put together to make it were odd, right? Three and three are each *odd*?
64	Shea:	Uh huh, and the other, the twos were even.
65	Ball:	**[A]** So you're kind of – I think Benny said then that he wasn't talking about *every* even number, right, Benny? **[B]** Were you saying that? **[C]** Some of the even numbers, like six, are made up of two odds, like you just suggested.
66	Benny:	Uh-uh (agreeing with the teacher).
67	Ball:	Other people's comments?